Math 201–Final Exam (Summer 15) B. Shayya and H. Yamani

- Please write your **section number** on your booklet.
- Please answer each problem on the **indicated page(s)** of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.
- The exam has a total of five problems. **Three of the problems are on the front side of this page and two are on the back side.**

Problem 1 (answer on pages **1**, **2**, **3**, and **4** of the booklet.)

(i) (9 pts) Find the work done by the field $F = x\,\mathbf{i} + y\,\mathbf{j} + z^3\,\mathbf{k}$ over the curve

$$C: \qquad r(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + 4t\,\mathbf{k}, \qquad 0 \le t \le 3.$$

(ii) (9 pts) Evaluate $\int_C 9x^2y\,dx + (1 + 2z^2 + 3x^3)\,dy + 4yz\,dz$, where C is any curve joining the point $(1, 2, 2)$ to the point $(1, 5, 2)$.

(iii) (9 pts) Use the curl test to find a function $h(z)$ that makes the differential form

$$(yz + \cos y + h(z))\,dx + (xz - x\sin y)\,dy + (xy + xh(z))\,dz$$

exact. Then find a potential function for it.

(iv) (9 pts) Let R be the region in the first quadrant enclosed by the parabola $y = x^2$, the line $y = 1$, and the y-axis. Use **Green's theorem** to find outward flux of the field $F(x, y) = y^2\,\mathbf{i} + 2xy^2\,\mathbf{j}$ around the boundary of R.

(v) (9 pts) Let the region R and the vector field F be the same as in part (iv). Find the outward flux around the boundary of R **directly**.

Problem 2 (answer on pages **5**, **6**, and **7** of the booklet.)

(a) (12 pts) Change $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)\,dy\,dx$ into an equivalent polar integral. Then evaluate the polar integral.

(b) (12 pts) Find the volume of the region D in space that is bounded from below by the xy-plane and from above by the paraboloid $z = 4 - x^2 - y^2$.

repeat last (c) (12 pts) Find the volume of the region G cut from the ball $\rho \le 1$ by the cone $\phi = \pi/3$.

Problem 3 (answer on pages **8**, **9**, and **10** of the booklet.)

(i) (15 pts) Find the normal line and tangent plane of the surface $z = x^2 + y^2$ at the point $(1, 1, 2)$.

(ii) (15 pts) Find the absolute maximum and minimum values of the function $f(x, y, z) = x + 2y + 3z$ on the sphere $x^2 + y^2 + z^2 = 1$.

(iii) (15 pts) Consider the function $f(x, y, z) = e^z \ln(xy)$. Estimate $f(e, 1.03, 0.04)$.

(45 pts)

Problem 4 (answer on pages **11**, **12** and **13** of the booklet.)
Let D be the region in space bounded from below by the plane $z = 0$, from above by the cone $z = 4 - \frac{\sqrt{x^2+y^2}}{3}$, and on the sides by the cylinder $x^2 + y^2 = 9$. Let B be the subset of D that is bounded by the planes $z = 0$ and $z = 3$. Also, let $f(x, y, z)$ be a function of three variables which is continuous on D.

(a) (9 pts) Write $\iiint_D f(x, y, z)dV(x, y, z)$ as an iterated triple integral(s) in cylindrical coordinates using the order of integration $dz\, dr\, d\theta$.

(b) (9 pts) Write $\iiint_D f(x, y, z)dV(x, y, z)$ as an iterated triple integral(s) in spherical coordinates using the order of integration $d\rho\, d\phi\, d\theta$.

(c) (9 pts) Write $\iiint_B f(x, y, z)dV(x, y, z)$ as an iterated triple integral(s) in spherical coordinates using the order of integration $d\phi\, d\rho\, d\theta$.

(d) (9 pts) Write $\iiint_D f(x, y, z)dV(x, y, z)$ as an iterated triple integral(s) in Cartesian coordinates using the order of integration $dx\, dz\, dy$.

(e) (9 pts) Write $\iiint_D f(x, y, z)dV(x, y, z)$ as an iterated triple integral(s) in Cartesian coordinates using the order of integration $dx\, dy\, dz$.

(9 pts)

Problem 5 (answer on page **14** of the booklet.)
(9 pts) One of the following three series converges. Find the convergent series and use one of the theorems you learned in the course to show that it indeed converges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n e^{1/n}}{n+1} \qquad \sum_{n=1}^{\infty} \frac{n^3 - n + 5}{\sqrt{n^7 + n^6 + n^5}} \qquad \sum_{n=1}^{\infty} \frac{e^{1/n} - 1}{n}$$